Read remark(2) below first.

**Definitions:**

$(E, d)$ be a metric space. $A, B \subseteq E$.

1. $B$ is dense in $E$ if the closure of $B$ equals $E$, i.e. $\overline{B} = E$
2. $A$ is nowheredense if the interior of $\overline{A}$ is empty, i.e. $(\overline{A})^o = \emptyset$.

**Claim:**

If $A$ is nowhere dense, then $E - A$ is dense in $E$.

The claim above look like to be true by the following observation.

**Observation:**

$(E, d)$ be a metric space. If $F, G \subseteq E$ such that $F \cap G = \emptyset$ and $E = F \cup G$, then $\overline{F} \cap G^o = \emptyset$ and $E = \overline{F} \cup G^o$.

**Proof:** (of the observation)
$\forall x \in E$,

1. either $\forall \rho > 0$, the open ball $B(x, \rho)$ contains at least one element of $F$, then $x \in \overline{F}$ by definition (but $x \notin G^o$ since $\forall \rho > 0$, $B(x, \rho)$ is not inside $G$);
2. or $\exists \rho > 0$ such that the open ball $B(x, \rho)$ contain no element of $F$, then $B(x, \rho) \subseteq G$ and $x \in G^o$, (obviously $x \notin \overline{F}$ in this case).

**Remarks:**

1. Set $F = E - \overline{A}$ and $G = \overline{A}$ in the above observation, we have $E = \overline{(E - \overline{A})} \cup (\overline{A})^o$. Thus, if $(\overline{A})^o$ is empty, then $E = \overline{(E - \overline{A})}$. Hence $E - \overline{A}$ is dense in $E$, and so is $E - A$ which contains $E - \overline{A}$
2. If we use '+' to denote disjoint union, then the above observation says $E = F + G \;\Rightarrow\; E = \overline{F} + G^o$. In this form, the claim can be easily seen to be true in the mind without going through the horribly complicated notations above.
3. It seems sometimes '$A$ is nowhere dense in $E$' is also defined by '$(E - \overline{A})$ is dense in $E$'.

**Reference:**

pp.134 and pp.159, Hausdorff,F. *Set Theory*, (Translated from the German by John R. Aumann, et al., Chelsea, 1991.