

Contents

Introduction	1
Chapter 1. Spin systems	3
1.1. Spin matrices	3
1.2. Rotation of spins and symmetries	5
1.3. Spin systems	6
1.4. Hamiltonians and their symmetries	8
1.5. Gibbs states and correlation functions	9
1.6. Phase diagrams of ferromagnetic models	10
1.7. Exercises	11
Chapter 2. Fermionic and bosonic systems	15
2.1. Fock spaces	15
2.2. Creation and annihilation operators	18
2.3. Bose–Einstein condensation	21
2.4. The Hubbard model	24
2.5. Spin operators and symmetries	26
2.6. Relation to the antiferromagnetic Heisenberg model	27
Chapter 3. Equilibrium States	33
3.1. States for finite systems	33
3.2. Gibbs states in Hilbert spaces	35
3.3. Infinite volume Gibbs states	37
3.4. Extremal Gibbs states	48
3.5. Decomposition of states	51
3.6. Exercises	53
Chapter 4. Uniqueness and non-uniqueness of Gibbs states	57
4.1. Uniqueness of KMS states at high temperature	57
4.2. Long-range order in the XXZ-model	60
4.3. Long-range order using infrared bounds	67
Chapter 5. Mean-field systems	71
5.1. Permutation-invariant states	71
5.2. The mean-field equation	74
5.3. Gibbs states of some mean-field models	76

5.4. Exercises	78
Chapter 6. 2D systems with continuous symmetry	79
Appendix A. Mathematical supplement	85
A.1. Matrices and matrix norms	85
A.2. Hölder inequality for matrices	85
A.3. Trotter and Duhamel	88
A.4. Further matrix inequalities	89
A.5. About convex functions	91
Appendix B. Solutions to some exercises	93
B.1. Spin systems	93
B.2. Fermionic systems	97
B.3. Equilibrium states	97
B.4. Uniqueness and non-uniqueness of Gibbs states	101
B.5. Mean-field systems	102
Appendix. Bibliography	103
Appendix. Index	105