Assignment 4

Due Monday 7 November 15:00. Two problems will be selected for marking among those with boxed numbers. There are **4 points** for each of the two problems, and **2 points** for presentation. Do not forget to write **your name** and **your group** (**1**: Tuesday 12-1/Buze; **2**: Tuesday 2-3/Vogel; **3**: Thursday 1-2/Khor; **4**: Friday 12-1/Matejczyk).

1. For which values of x is the Gamma function $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$ Riemann-integrable?

2. Find the following integrals.

1. $\displaystyle\int_0^1 \frac{\cos\sqrt{t}}{\sqrt{t}}\, dt$.
2. $\displaystyle\int_2^\infty t^{-2} \log t\, dt$.
3. $\displaystyle\int_2^\infty e^{-\sqrt{t}} dt$.
4. $\displaystyle\int_0^\pi e^{\sin^2 t} \sin t \cos t\, dt$.

3. Let $f(x) = \displaystyle\int_x^{x^2} e^{-t^2} dt$. Find its derivative $f'(x)$, and draw it for $x \in (-\infty, \infty)$. For approximately which x is f maximum?

4. Find the pointwise limits of the following functions as $n \to \infty$. Is the convergence uniform? Prove it!

1. $f_n(x) = x^{1/n}$ for $x \in [0, 1]$.
2. $f_n(x) = \sin(x + \frac{1}{n})$ for $x \in \mathbb{R}$.
3. $f_n(x) = e^{n(\cos x - 1)}$ for $x \in \mathbb{R}$.
4. $f_n(x) = e^{x/n}$ for $x \in [0, 2\pi]$.

5. Find the pointwise limits of the following functions as $n \to \infty$. Is the convergence uniform? Prove it!

1. $f_n(x) = \min(\cos x, 1 - \frac{1}{n})$ for $x \in \mathbb{R}$.
2. $f_n(x) = n \sin \frac{x}{n}$ for $x \in \mathbb{R}$.
3. $f_n(x) = e^{-x/n}$ for $x \in [0, \infty)$.
4. $f_n(x) = \lim_{m\to\infty} [\cos(n!\pi x)]^{2m}$ for $x \in [0, 1]$.

6. Consider the functions $f_n(x) = n^a x\, e^{-n^b x}$ on $[0, \infty)$, where a, b are fixed numbers. Find the pointwise limit as $n \to \infty$. Draw a few functions, calculate the derivatives, and find the values of a, b for which the convergence is uniform.