Assignment 3

Due Monday 31 October 15:00. Two problems will be selected for marking among those with boxed numbers. There are **4 points** for each of the two problems, and **2 points** for presentation. Do not forget to write **your name** and **your group** (**1**: Tuesday 12-1/Buze; **2**: Tuesday 2-3/Vogel; **3**: Thursday 1-2/Khor; **4**: Friday 12-1/Matejczyk).

1. Find the derivatives of the following functions:

1. $F(x) = \int_1^x \log t \, \mathrm{d}t, x > 1;$
2. $G(x) = \int_x^0 \sqrt{1+t^4} \mathrm{d}t, x \in \mathbb{R};$
3. $H(x) = \int_x^{x^2} \mathrm{e}^{-t^2} \mathrm{d}t, x \in \mathbb{R};$
4. $I(x) = \int_{\frac{1}{x}}^{\sqrt{x}} \cos(t^2) \mathrm{d}t, x > 0.$

(Hint: Use the chain rule and the fundamental theorem of calculus.)

2. Find the following integrals.

1. $\int_0^1 \log(1+x) \, \mathrm{d}x;$
2. $\int_{-2}^{-1} \frac{1}{x^3} \, \mathrm{d}x;$
3. $\int_{-a}^{a} \mathrm{e}^t \, \mathrm{d}t$ for $a \in \mathbb{R};$
4. $\int_0^a t \, \cos(t^2) \, \mathrm{d}t$ for $a > 0.$

3. Evaluate the following improper integrals if they are convergent (or establish their divergence).

1. $\int_0^1 \log x \, \mathrm{d}x;$
2. $\int_0^1 \frac{1}{\sin x} \, \mathrm{d}x;$
3. $\int_0^\infty \sin t \, \mathrm{d}t;$
4. $\int_0^\infty \mathrm{e}^{\cos t} \sin t \, \mathrm{d}t.$

4. Prove the Cauchy-Schwarz inequality for regulated functions. Namely, if $f, g \in R[a, b]$, prove that

$$\Big(\int_a^b f(x)g(x) \mathrm{d}x \Big)^2 \leq \int_a^b f(x)^2 \mathrm{d}x \int_a^b g(x)^2 \mathrm{d}x.$$

(Hint: You can try a proof similar to that of Problem 4 in Assignment 1.)

5. Show that the set of discontinuities of $f \in R[a, b]$ is countable. (Hint: Check that the set of jumps of size $\frac{1}{n}$ is finite.)