15% of the credit for this module will come from your work on eight assignments. Each assignment will be marked out of 25 for answers to one randomly chosen 'A' and one 'B' questions. Working through all questions is vital for understanding lecture material and success at the exam. 'A' questions will constitute a base for the first exam problem worth 40% of the final mark, the rest of the exam will be based on 'B' questions.

The answers to **all questions** are to be submitted by the deadline of **3pm on Monday 3 November 2014**. Your work should be stapled together, and you should state legibly at the top your name, your department and the name of your teaching assistant. Your work should be deposited in the dropbox labelled with your teaching assistant's name, opposite the Maths Undergraduate Office.

Properties of regulated functions.

1. B. Let $f : [0, 2] \to \mathbb{R}$ be a regulated function. Let $(x_n)_{n\geq 1}$ be a sequence in $[0, 1)$ with $\lim_{n\to\infty} x_n = 1$. Prove that the sequence $(f(x_n))_{n\geq 1}$ is Cauchy.

Improper integrals

2. A. Calculate $\lim_{R\to\infty} \int_{-R}^{R} x dx$. Does your result imply that $\int_{-\infty}^{\infty} x dx$ exists? Explain your answer.
3. B. Evaluate the improper integrals (or establish their divergence):
   (a) $\int_0^1 log(t)dt$;
   (b) $\int_0^1 \frac{1}{t^p} dt,\ p > 0$;
   (c) $\int_1^\infty \frac{1}{t^p} dt,\ p > 0$;
   (d) $\int_0^\infty cos(x)dx$.

Uniform convergence and uniform continuity.

4. A. Let $A \subset \mathbb{R}$ and let $f, g : A \to \mathbb{R}$ be uniformly continuous. Prove that $f + g : A \to R$ is uniformly continuous.
5. B. If $a < b$ and $f : (a, b) \to \mathbb{R}$ is uniformly continuous, show that f is bounded. Give examples of functions $f, g : \mathbb{R} \to \mathbb{R}$ which are uniformly continuous such that f is not bounded but g is bounded.
6. B. If f is regulated and is the uniform limit of weakly increasing step functions show that f is weakly increasing. [f is *weakly increasing* means $u < v \Rightarrow f(u) \leq f(v)$.]