**MA258 Analysis III** **Assignment 3.**

*15% of the credit for this module will come from your work on eight assignments. Each assignment will be marked out of 25 for answers to one randomly chosen 'A' and one 'B' questions. Working through all questions is vital for understanding lecture material and success at the exam. 'A' questions will constitute a base for the first exam problem worth 40% of the final mark, the rest of the exam will be based on 'B' questions.*

The answers to **all questions** are to be submitted by the deadline of **3pm on Monday 27 October 2014**. Your work should be stapled together, and you should state legibly at the top your name, your department and the name of your teaching assistant. Your work should be deposited in the dropbox labelled with your teaching assistant's name, opposite the Maths Undergraduate Office.

1. A. Give an example of a regulated function $f : [a, b] \to \mathbb{R}$ with the properties that $\forall x \in [a, b]\ f(x) \geq 0$, $\int_a^b f = 0$ and there is $c \in [a, b]$ with $f(c) > 0$. **For glory (but not for credit):** Find $f$ satisfying all of the above, which is NOT a step function.

2. A. Show that a continuous function $f : [a, b] \to \mathbb{R}$ with the properties that $\forall x \in [a, b]\ f(x) \geq 0$ **and** $\int_a^b f = 0$ must be identically zero.

3. A. Find the points of the extremum of the function

$$f(x) = \int_1^x \frac{sin(t)}{t} dt$$

in the region $x > 1$.

4. A. Find the derivatives of the following functions:

   (a) $F(x) = \int_1^x log(t)dt,\ x > 1$;

   (b) $G(x) = \int_x^0 \sqrt{1 + t^4}dt,\ x \in \mathbb{R}$;

   (c) $H(x) = \int_x^{x^2} e^{-t^2}dt,\ x \in \mathbb{R}$;

   (d) $I(x) = \int_{\frac{1}{x}}^{\sqrt{x}} cos(t^2)dt,\ x > 0$.

   **Hint.** Use the chain rule, the FTC and the following two facts: $\int_a^b f = -\int_b^a f$ and $\int_a^b f = \int_a^c f + \int_c^b f$.

5. B. Applying the second version of the fundamental theorem of calculus (aka Newton-Leibnitz formula) find the following integrals:

   (a) $\int_0^1 log(1 + x)dx$;

   (b) $\int_{-2}^{-1} \frac{1}{x^3}dx$;

   (c) $\int_{-x}^{x} e^t dt,\ x \in \mathbb{R}$;

   (d) $\int_0^x t \cdot cos(t^2)dt,\ x > 0$.

6. B. Define $f : [0,1] \to \mathbb{R},\ f(x) := 0$ if $x \notin \mathbb{Q},\ f(p/q) := 1/q, q > 0,\ p, q$ coprime integers.
   (i) Show that $f$ is continuous at $1/\sqrt{2}$ but not at $1/3$.
   (ii) Prove that $f \notin S[0,1]$.
   (iii)Prove that $f$ is regulated.
   **Hint.** For any $\varepsilon > 0$ construct a step function $\varphi : [0,1] \to \mathbb{R}$ such that $\|f - \varphi\|_\infty < \varepsilon$. Restrict to $q < 1/\varepsilon$.

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